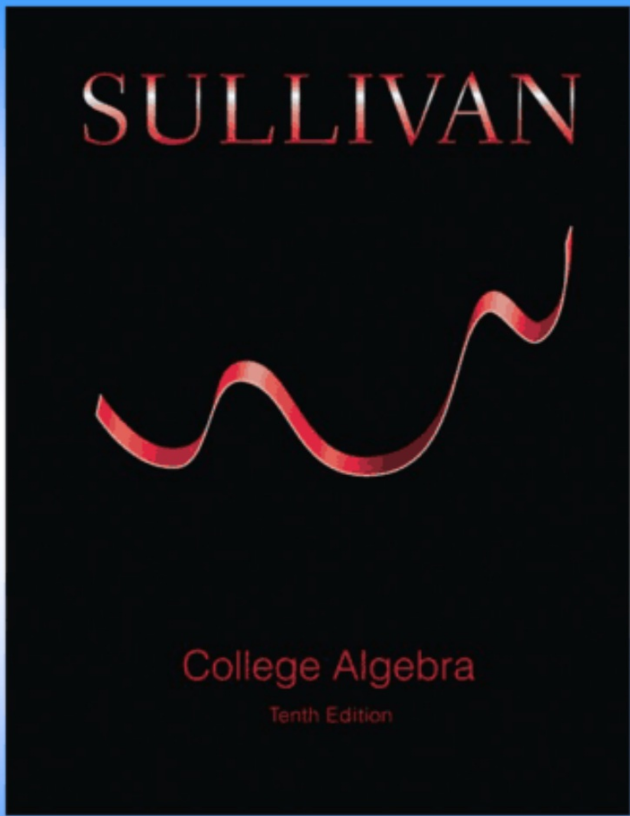
Chapter 3

Section 1



3.1 Functions

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Intervals (Section 1.5, pp. 120–121)
- Solving Inequalities (Section 1.5, pp. 123–126)
- Evaluating Algebraic Expressions, Domain of a Variable (Chapter R, Section R.2, pp. 20–23)
- Rationalizing Denominators (Chapter R, Section R.8, p. 75)

Now Work the **'Are You Prepared?'** problems on page 210.

OBJECTIVES
1. Determine Whether a Relation Represents a Function (p. 199)
2. Find the Value of a Function (p. 202)

4. Find the Domain of a Function Defined by an Equation (p. 206)
5. Form the Sum, Difference, Product, and Quotient of Two Functions (p. 208)

Definitions

Relation – any equation that shows the relationship between two variables x, y.

Function – a relation in which each x is paired with exactly one y.

Domain – the set of all possible x-values (input, independent variable)

Range – the set of all possible y-values (output, dependent variable)

Example

Determining Whether a Relation Is a Function

For each relation in Figures 6, 7, and 8, state the domain and range. Then determine whether the relation is a function.

(a) See Figure 6. For this relation, the input is the number of calories in a fast-food sandwich, and the output is the fat content (in grams).

D: $\{580, 650, \ldots\}$

R: $\{31, 37, \ldots\}$

It is a Function.

Figure 6

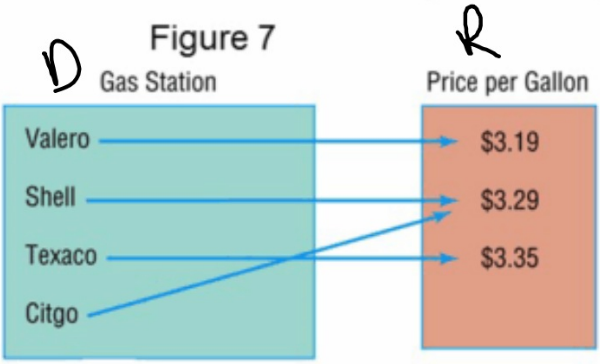
Source: Each company's Web site

Example continued

(b) See Figure 7. For this relation, the inputs are gasoline stations in Harris County, Texas, and the outputs are the price per gallon of unleaded regular in March 2014.

(c) See Figure 8. For this relation, the inputs are the weight (in carats) of pear-cut diamonds and the outputs are the price (in dollars).

Figure 7



Function

Figure 8

Source: Used with permission of Diamonds.com

Not a function

Example

Determining Whether a Relation Is a Function

For each relation, state the domain and range. Then determine whether the relation is a function.

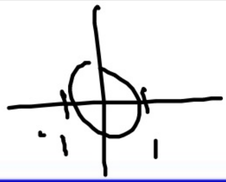
(a) $\{(1, 4), (2, 5), (3, 6), (4, 7)\}$
(b) $\{(1, 4), (2, 4), (3, 5), (6, 10)\}$
(c) $\{(-3, 9), (-2, 4), (0, 0), (1, 1), (-3, 8)\}$

(a) D: $\{1,2,3,4\}$
R: $\{4,5,6,7\}$
Function

(b) D: $\{1,2,3,6\}$
R: $\{4,5,10\}$
Function

(c) D: $\{-3,-2,0,1\}$
R: $\{9,4,0,1,8\}$
Not a function

Example



Determining Whether an Equation Is a Function

Determine whether the equation $x^2 + y^2 = 1$ defines y as a function of x.

If you recognize that $x^2+y^2=1$ is a circle, then you know it is NOT a function.

x	y
-1	0
0	±1 *

not a function

$(-1)^2+y^2=1$

$y^2=0$

$y=0$

$0^2+y^2=1$

$y^2=1$

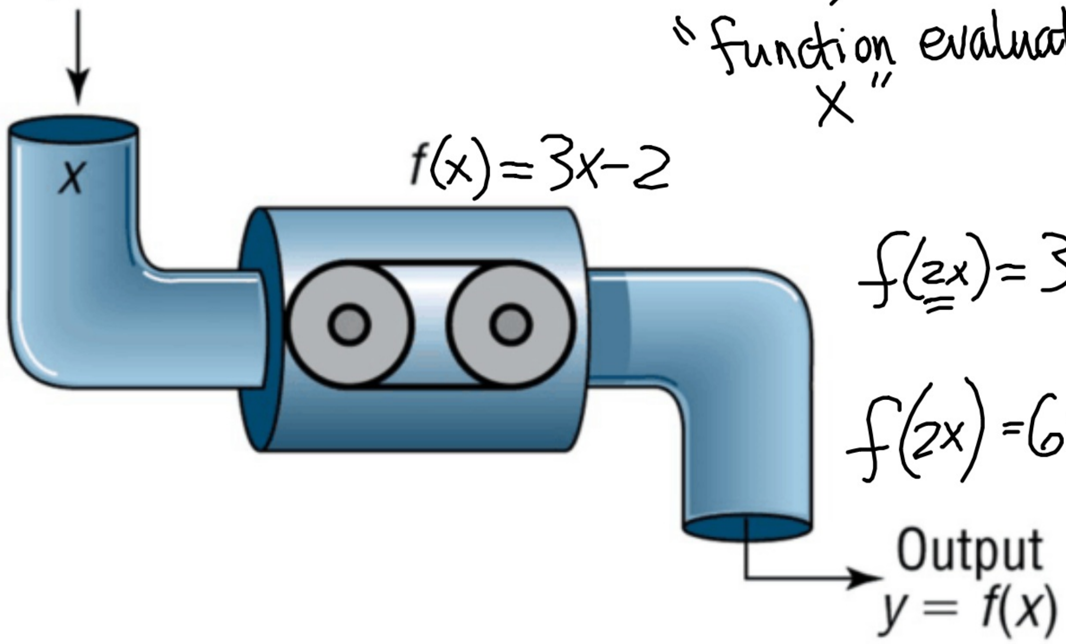
$y=\pm 1$

Figure: Input/Output Machine

Input $x = 2x$ function notation: $f(x)$

"function evaluated at x"

$f(x) = 3x - 2$



$f(\underline{\underline{2x}}) = 3(\underline{\underline{2x}}) - 2$

$f(2x) = 6x - 2$

Example

Finding Values of a Function

For the function f defined by $\boxed{f(x) = 2x^2 - 3x,}$ evaluate

(a) $f(3)$ (b) $f(x) + f(3)$ (c) $3f(x)$ (d) $f(-x)$
(e) $-f(x)$ (f) $f(3x)$ (g) $f(x + 3)$ *(h)* $f(x + h)$

(a) $f(3) = 2(3)^2 - 3(3) = 9$

(b) $\underbrace{f(x)} + \underbrace{f(3)}$

$\boxed{2x^2 - 3x + 9}$

(c) $3 \cdot \underbrace{f(x)}$

$3(2x^2 - 3x)$

$\boxed{6x^2 - 9x}$

d) $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

e) $-f(x) = -(2x^2 - 3x) = -2x^2 + 3x$

f) $f(3x) = 2(3x)^2 - 3(3x) = 18x^2 - 9x$

g) $f(x+3) = 2(x+3)^2 - 3(x+3)$

$= 2(x^2 + 6x + 9) - 3(x+3)$

$= 2x^2 + 12x + 18 - 3x - 9$

$= 2x^2 + 9x + 9$

(h) $f(x+h) = 2(x+h)^2 - 3(x+h)$

$$= 2(x^2+2xh+h^2) - 3(x+h)$$

$$= \boxed{2x^2+4xh+2h^2-3x-3h}$$

Finding the Domain of a Function Defined by an Equation

1. Start with the domain as the set of real numbers.
2. If the equation has a denominator, exclude any numbers that give a zero denominator.

denom $\neq 0$

3. If the equation has a radical of even index, exclude any numbers that cause the expression inside the radical (the radicand) to be negative.

radicand ≥ 0

Example

Finding the Domain of a Function

Find the domain of each of the following functions.

(a) $f(x) = x^2 + 5x$

(b) $g(x) = \dfrac{3x}{x^2 - 4}$

(c) $h(t) = \sqrt{4 - 3t}$

(d) $F(x) = \dfrac{\sqrt{3x + 12}}{x - 5}$

(a) D: all real numbers
$(-\infty, \infty)$

(b) D: $x^2 - 4 \neq 0$
$x^2 \neq 4$
D: $x \neq \pm 2$

(c) $h(t) = \sqrt{4-3t}$

$D: 4-3t \geq 0$

$-3t \geq -4$

$\boxed{D: t \leq 4/3}$

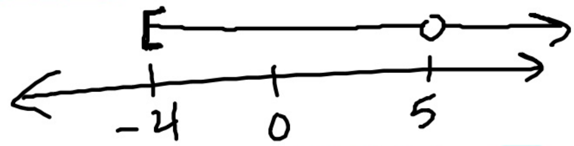
$\boxed{D: \left(-\infty, 4/3\right]}$

(d) $F(x) = \dfrac{\sqrt{3x+12}}{x-5}$

$3x+12 \geq 0$; $x-5 \neq 0$

$3x \geq -12$

$\boxed{D: x \geq -4 \ ; x \neq 5}$



$\boxed{D: [-4, 5) \cup (5, \infty)}$

Definition

If f and g are functions:
The **sum** $\boldsymbol{f + g}$ is the function defined by

$$(f + g)(x) = f(x) + g(x)$$

The **difference** $\boldsymbol{f - g}$ is the function defined by

$$(f - g)(x) = f(x) - g(x)$$

Definition

The **product** $\boldsymbol{f \cdot g}$ is the function defined by

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

The **quotient** $\boldsymbol{\frac{f}{g}}$ is the function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0$$

Example

Find all four function operations for $f(x) = 5x - 2$ and $g(x) = x^2 - 2x - 1$. State the domain of each.

$$\overset{f}{} \qquad \overset{g}{}$$

a) $f + g = (5x-2) + (x^2-2x-1) = x^2+3x-3$

b) $f - g = (5x-2) - (x^2-2x-1) = -x^2+7x-1$

c) $fg = (5x-2)(x^2-2x-1)$

$= 5x^3-10x^2-5x$
$\quad -2x^2+4x+2$

$= 5x^3-12x^2-x+2$

for a) b) and c) the domain is all reals

d) $\frac{f}{g} = \frac{5x-2}{x^2-2x-1}$

$D: \ x^2-2x-1 \neq 0$

$x \neq \frac{2 \pm \sqrt{4-4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

$= \frac{\cancel{2}}{2} \pm \frac{2\sqrt{2}}{2}$

$\boxed{D: \ x \neq 1 \pm \sqrt{2}}$

Application

Suppose that the revenue *R*, in dollars, from selling *x* clocks is $R(x) = 30x$. The cost *C*, in dollars, of selling *x* clocks is $C(x) = 0.1\,x^2 + 7x + 400$.

a. Find the profit function, $P(x) = R(x) - C(x)$

$P(x) = 30x - (0.1x^2 + 7x + 400)$

b. Find and interpret $P(30)$. $P(x) = -0.1x^2 + 23x - 400$

$P(30) = -0.1(30)^2 + 23(30) - 400$

$P(30) = 200 \rightarrow (30, 200) \rightarrow$ If we sell 30 clocks the profit is \$200